Acknowledgment

The authors wish to thank Professor A. K. Ghatak for his constant encouragement.

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The Interdigitated Three-Strip Coupler

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Abstract —**A general design procedure for three-strip interdigitated couplers with arbitrary coupling values is presented. These results are then applied, for various coupling values, to both stripline and microstrip media to check the physical realizability. The dimensions of the coupler can be substantially affected by allowing a small degree of impedance mismatching.**

I. Introduction

Interdigitated three-line microstrip couplers have been described in the literature [1], [2]. Their dimensions were arrived at in a form which required rather complicated mathematical manipulations and optimization techniques. The approach taken here is to develop the capacitance matrix for the generalized three-line interdigitated coupler. Using this matrix, the parameters required to provide any degree of coupling in any type of media (inhomogeneous microstrip or homogeneous stripline) are easily derived.

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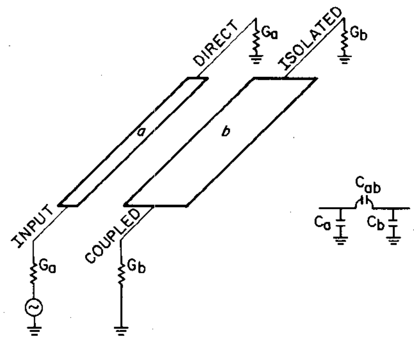

Fig. 1. Unequal width two-line coupler.

II. Asymmetrical Coupled Lines in Homogeneous Media

The construction of three-line couplers generally requires knowledge of the electrical characteristics of asymmetrical coupled lines. Cristal introduced the concept of unequal odd-mode and even-mode admittances for each individual line of a set of coupled lines of unequal widths in a homogeneous media [3]. The electrical characteristics of the coupled-line directional coupler are completely specified by either the capacitances per unit length or the odd and even mode admittances. The relationship between the coupler parameters, the coupling ratio and terminating admittances, and the line parameters as developed by Cristal are summarized in the following discussion.

Fig. 1 shows the coupler formed by line a and line b, both of which have an electrical length of ninety degrees. The odd and even mode admittances of each line are related to the per unit length mutual capacitance C_{ab} and the self capacitances C_a and C_b. These admittances are also related to the voltage coupling coefficient k and the respective terminating admittances G_a and G_b. The odd and even mode admittances can be expressed as

line a

$$Y_{oe}^{(a)} = vC_a = \frac{G_a - k\sqrt{G_aG_b}}{\sqrt{1-k^2}} \tag{1}$$

$$Y_{oo}^{(a)} = 2vC_{ab} + vC_a = \frac{G_a + k\sqrt{G_aG_b}}{\sqrt{1-k^2}} \tag{2}$$

line b

$$Y_{oe}^{(b)} = vC_b = \frac{G_b - k\sqrt{G_aG_b}}{\sqrt{1-k^2}} \tag{3}$$

$$Y_{oo}^{(b)} = 2vC_{ab} + vC_b = \frac{G_b + k\sqrt{G_aG_b}}{\sqrt{1-k^2}} \tag{4}$$

in which v is the velocity of propagation in the line media. Also note that $Y_{oo}^{(a)}$ is the odd-mode admittance of line a with respect to line b and $Y_{oo}^{(b)}$ is the odd-mode admittance of line b with respect to line a. If the width of each line in Fig. 1 were made equal, the coupler becomes symmetric with equal self capacitances C and equal terminating admittances G_o.

III. The Three-Line Coupler in Homogeneous Media

The general three-line coupler is formed by lines 1, 2, and 3 as shown in Fig. 2 together with the self and mutual capacitance representation. Lines 1 and 3 are tied on both ends to form a four-port coupler structure. For the same desired coupler perfor-

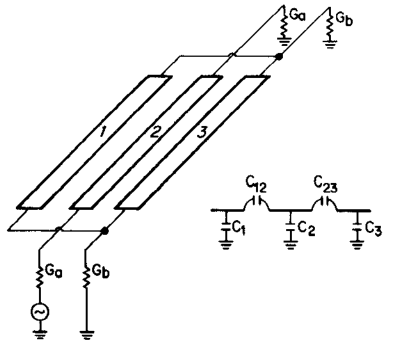

Fig. 2. Three-line coupler.

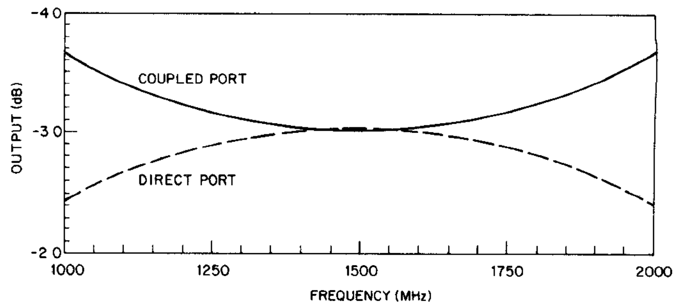

Fig. 3 Gain response of 3-dB three-line coupler.

mance parameters k, G_a, and G_b, the three-line asymmetric coupler and the two-line asymmetric coupler must have identical capacitance matrices. Therefore, the sum of the self capacitances of lines 1 and 3 must be equal to the self capacitance C_b of the wider line of the two-line case, and the sum of the new coupling capacitances must equal the two-line coupling capacitance C_{ab}

$$C_1 + C_3 = C_b \tag{5}$$

$$C_{12} + C_{23} = C_{ab} \tag{6}$$

$$C_2 = C_a. \tag{7}$$

The odd- and even-mode admittances for each pair of lines may be expressed in terms of the coupling ratio and terminal admittances. The additional assumption that the widths of line 1 and of line 3 are equal simplifies mathematical expressions and physical realizability but does not reduce generality, thus, by direct substitution of (1)–(4), (7), and

$$C_{12} = C_{23} = \frac{C_{ab}}{2} \tag{8}$$

$$C_1 = C_3 = \frac{C_b}{2} \tag{9}$$

we obtain

$$Y_{oe}^{(1)} = Y_{oe}^{(3)} = \frac{vC_b}{2} = \frac{G_a - k\sqrt{G_a G_b}}{2\sqrt{1-k^2}} \tag{10}$$

$$Y_{oo}^{(1)} = Y_{oo}^{(3)} = vC_{ab} + \frac{vC_b}{2} = \frac{G_a + k\sqrt{G_a G_b}}{2\sqrt{1-k^2}} \tag{11}$$

$$Y_{oe}^{(2)} = vC_a = \frac{G_b - k\sqrt{G_a G_b}}{\sqrt{1-k^2}} \tag{12}$$

$$Y_{oo}^{(2)} = vC_{ab} + vC_a = \frac{G_b}{\sqrt{1-k^2}}. \tag{13}$$

Equations (10)–(13) establish the required line parameters needed to produce a three-line coupler with an arbitrary degree of coupling. When equating the capacitance matrix of an electrically symmetric two-line coupler, the equivalent three-line coupler will also be electrically symmetric ($G_a = G_b$).

With the aid of (10)–(13), the line admittances for a 3-dB coupler with $G_a = G_b = G_o$ in a 50-Ω system ($G_o = 20$ mS) were calculated as

$$Y_{oe}^{(1)} = Y_{oe}^{(3)} = 4.134 \text{ mS} \tag{14}$$

$$Y_{oo}^{(1)} = Y_{oo}^{(3)} = 24.180 \text{ mS} \tag{15}$$

$$Y_{oe}^{(2)} = 8.272 \text{ mS} \tag{16}$$

$$Y_{oo}^{(2)} = 28.316 \text{ mS}. \tag{17}$$

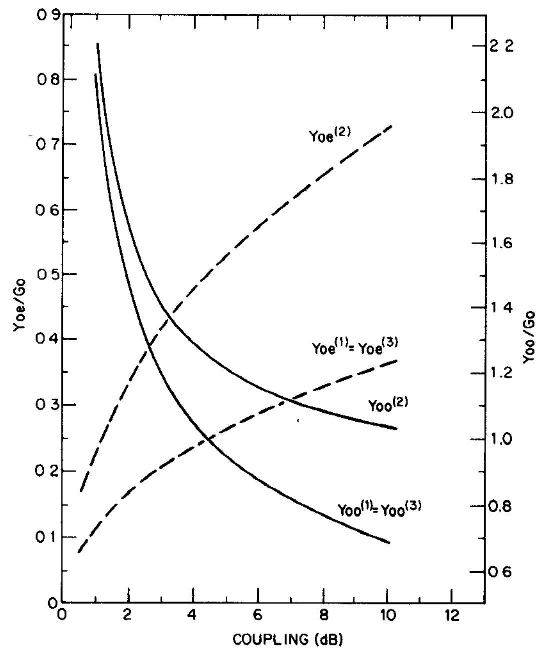

Fig. 4. Mode admittances of three-line coupler for arbitrary coupling ratios.

The performance of the coupler with these line parameters was checked by using a general linear circuit analysis program called ANA, which is capable of analyzing multiconductor-coupled-line circuits, as well as lumped-element RLC networks [4]. The calculated gain response is shown in Fig. 3. The input and output VSWR were unity at all ports and the isolation was in excess of 60 dB.

The odd- and even-mode admittances for arbitrary amounts of coupling for the case in which $G_a = G_b = G_o$ can be calculated in a similar manner. Fig. 4 shows a plot of the odd- and even-mode admittances normalized to the couplers characteristic admittance G_o as a function of coupling values in decibels.

IV. Mismatched Couplers

In many cases, the desired coupling ratio and terminating admittances along with the physical dimensions of the substrate require line and gap widths which are unrealistically narrow. Here it will be shown that a small allowable mismatch at each port increases both the even-mode admittance of the end line and the resulting physical dimensions.

The VSWR at all ports will be the same if

$$r = \sqrt{\text{VSWR}} = \frac{G_a}{G_o} = \frac{G_o}{G_b} \tag{18}$$

TABLE I
A SUMMARY OF LINE PARAMETERS

MEDIA	ELECTRICAL SYMMETRY	ADMITTANCES				MODE CONSTANTS			
Inhomogeneous	non-sym	Yca	Ycb	Ypa	Ypb	γc	γp	Rc	R
Homogeneous	non-sym	Yca	Ycb	Ypa	Ypb	γ	γ	Rc	-Rc
		Yea	Yeb	Yoa	Yob	γ	γ	1	-1
Inhomogeneous	sym	Ye	Ye	Yo	Yo	γe	γo	1	-1
Homogeneous	sym	Ye	Ye	Yo	Yo	γ	γ	1	-1

TABLE II
DIMENSIONS FOR $\epsilon = 10$ MICROSTRIP

coupl dB	W1/H	W2/H	S1/H	(1) Yc	(1) Yp	(2) Yc	(2) Yp	Rc	Rp	ϵc	ϵp
2.7	.006	0.751	0.009	.000	14.27	10.83	>1E6	1.381	-.000	7.973	5.994
3.0	.017	0.798	0.022	.140	14.25	11.12	1135	1.398	-.009	7.969	5.836
6.0	.063	1.097	0.183	2.16	12.12	13.74	77.06	1.620	-.097	8.046	5.563
8.34	.082	1.165	0.363	3.35	11.33	15.20	51.41	1.803	-.122	7.817	5.516
10.	.092	1.185	0.510	4.16	10.98	16.01	42.27	1.917	-.136	7.664	5.503
20.	.134	1.200	1.650	7.62	10.22	18.68	25.07	2.428	-.168	7.181	5.596

in which

$$G_o = \sqrt{G_a G_b}. \tag{19}$$

The odd- and even-mode line admittances can be expressed in terms of allowable VSWR

$$Y_{oe}^{(1)} = Y_{oe}^{(3)} = \frac{G_o(r-k)}{2\sqrt{1-k^2}} \tag{20}$$

$$Y_{oo}^{(1)} = Y_{oo}^{(3)} = \frac{G_o(r+k)}{2\sqrt{1-k^2}} \tag{21}$$

$$Y_{oe}^{(2)} = \frac{G_o(1/r-k)}{\sqrt{1-k^2}} \tag{22}$$

$$Y_{oo}^{(2)} = \frac{G_o/r}{\sqrt{1-k^2}}. \tag{23}$$

Even though the coupling is not affected by the mismatch, the power transmitted to the two-output ports and the isolation are slightly modified.

V. CONSIDERATIONS OF PHYSICAL REALIZABILITY

The ability to fabricate a two-line 3-dB coupler in microstrip with standard thin-film lithography is marginal. In applications requiring a coupling range between 3 and 6 dB, the three-line-coupler design provides more easily realizable physical dimensions.

The relationship between the physical characteristics of asymmetric coupled lines (width, spacing, dielectric constant, etc.) and their electrical properties have been investigated using many approaches [5]–[9]. The technique used by Bedair, which seems to be the simplest both mathematically and conceptually, provides excellent results [10]. This technique derives the total self and mutual capacitance of each line by first calculating their component capacitances. The expression for each of the component capacitances as derived by conformal mapping or empirical techniques by previous investigators have been used. The results are closed-form solutions for both coupled stripline and microstrip lines of unequal widths. Here the physical line dimensions for the three-line coupler are arrived at by adapting this technique to the three-line case.

The electrical parameters of coupled asymmetric lines in an inhomogeneous media (e.g., microstrip) cannot be accurately specified using odd- and even-mode characteristics, except for some specific cases. In general, they can be represented by two orthogonal modes called the pi and c modes [11]. Table I indicates under what conditions the use of pi and c modes or odd and even modes are appropriate. Note that the different modes have different velocities, characterized by the propagation constants, only in the inhomogeneous case. It is also important to realize that electrical symmetry is not the same as physical symmetry with the exception of the simple two-line case. When three lines are being considered and they are all of equal widths, the fringing capacitance on the outside edge of the end lines differs from the fringing capacitance between the lines, resulting in electrical asymmetry even though there exists physical symmetry.

It is interesting to note that, for the case of homogeneous asymmetric lines, an example of which is the three-line coupler in stripline, the line parameters may be expressed in pi–c mode form or even–odd mode form. They are electrically equivalent.

The coupler design procedure consists of first calculating the odd- and even-mode line admittances of all lines for the desired coupling ratio. In the inhomogeneous case, the odd- and even-mode parameters represent the equivalent pi and c mode parameters, as if the propagation constants for each mode were the same. Bedair's method is then adapted to obtain the physical dimensions, and in the case of microstrip, the c and pi mode parameters.

This technique was used to calculate the physical dimensions for 50-Ω matched couplers with coupling values ranging between 2.7 and 6 dB for a microstrip configuration on a substrate having a dielectric constant of 10. These values, along with the calculated pi and c mode line parameters, are shown in Table II. The dimensions for coupling values around 3 dB are very difficult to achieve with small dielectric heights. Practical production limits for thin-film lines and gaps on alumina substrates are about 1 mil. For a 3-dB coupler on a 25-mil substrate, the line widths are 0.43 mil and the gap spacing is 0.55 mil. These dimensions still pose a realization problem. Allowing a slight mismatch will overcome this problem.

Equations (20)–(23) were used in calculating the coupler dimensions for a 2.7- and 3-dB coupler. These dimensions as a function of allowable VSWR are shown in Figs. 5 and 6 for the 2.7- and 3.0-dB couplers, respectively. This range of coupling covers all nominal 3-dB couplers with bandwidths up to an

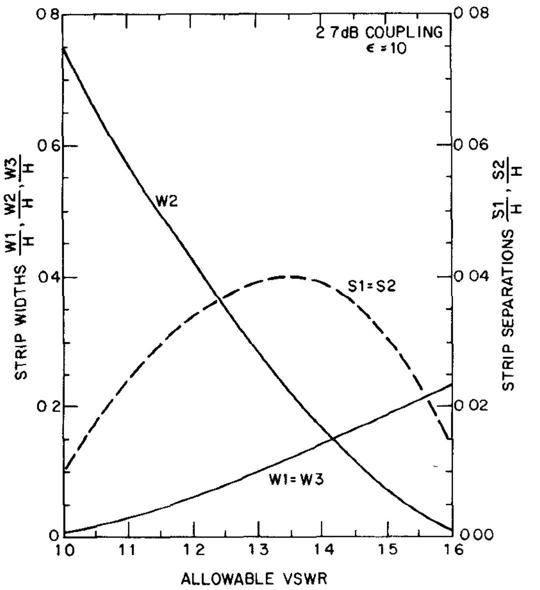

Fig. 5. Dimensions for the mismatched 2.7-dB coupler.

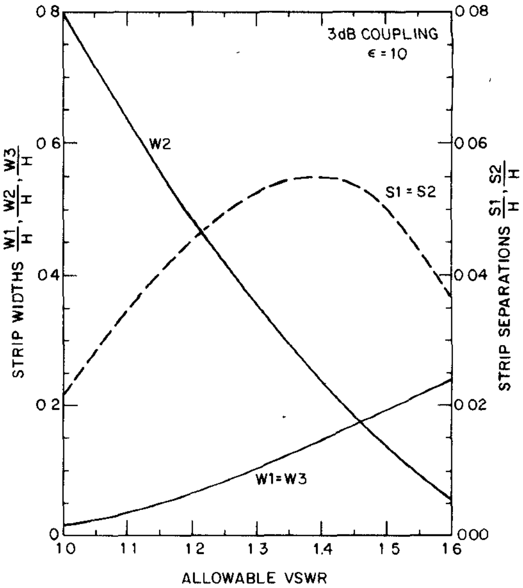

Fig 6 Dimensions for the mismatched 3.0-dB coupler.

TABLE III
RESPONSE OF THE 2.7-dB COUPLER

Freq(gHz)	Direct(dB)		Coupled(dB)		Isolated(dB)		Reflection line a		Coefficient line b	
4.5	-2.74	-69.0	-3.40	22.0	-30.9	-178	.098	6	.077	-138
5.0	-2.98	-74.2	-3.13	16.9	-30.9	180	.099	0	.082	-143
5.5	-3.17	-79.1	-2.95	12.1	-30.8	179	.099	-6	.087	-147
6.0	-3.30	-83.8	-2.83	7.5	-30.6	179	.099	-12	.091	-151
6.5	-3.37	-88.4	-2.77	3.0	-30.2	180	.098	-18	.094	-155
6.75	-3.38	-90.8	-2.76	0.8	-29.9	-180	.097	-22	.096	-157
7.0	-3.37	-93.1	-2.76	-1.5	-29.6	-179	.096	-25	.098	-159
7.5	-3.31	-97.7	-2.81	-6.0	-28.7	-178	.093	-32	.102	-162
8.0	-3.19	-102.0	-2.92	-10.5	-27.6	-178	.090	-40	.106	-165
8.5	-3.01	-107.0	-3.10	-15.3	-26.3	-178	.086	-50	.111	-169
9.0	-2.77	-113.0	-3.36	-20.3	-24.9	180	.081	-61	.116	-173

TABLE IV
RESPONSE OF THE 2.7-dB COUPLER ASSUMING EQUAL MODE VELOCITIES

Freq(gHz)	Direct(dB)		Coupled(dB)		Isolated(dB)		Reflection line a		Coefficient line b	
4.5	-2.76	-68.7	-3.34	21.3	-65.3	133	.085	21	.084	-159
5.0	-3.00	-73.7	-3.08	16.3	-65.3	123	.087	16	.086	-164
5.5	-3.19	-78.5	-2.90	11.5	-65.3	113	.089	12	.088	-168
6.0	-3.32	-83.2	-2.79	6.8	-65.3	104	.090	7	.090	-173
6.5	-3.38	-87.7	-2.73	2.3	-65.3	95	.091	2	.090	-178
6.75	-3.39	-90.2	-2.73	-.2	-65.3	89	.091	0	.090	-178
7.0	-3.38	-92.3	-2.73	-2.3	-65.3	85	.091	-2	.090	177
7.5	-3.32	-96.8	-2.79	-6.8	-65.3	76	.090	-7	.090	173
8.0	-3.19	-101.0	-2.90	-11.5	-65.3	67	.089	-11	.088	169
8.5	-3.00	-106.0	-3.08	-16.3	-65.3	57	.087	-16	.086	164
9.0	-2.76	-111.0	-3.34	-21.3	-65.3	47	.085	-21	.084	159

octave. Fig. 5 indicates that the gap spacing is the smallest for a 2.7-dB coupler and to achieve a 1-mil spacing on 25-mil substrate the allowable VSWR must be on the order of 1.3:1. By increasing the thickness to 50 mils, the allowable VSWR reduces to 1.07:1 for the same spacing of 1 mil. The 3-dB coupler reaches a 1-mil-wide line and gap at nearly the same value of allowable VSWR (1.12:1 to 1.14:1) on a 25-mil substrate.

The dimensions for a 2.7-dB coupler with a 1.2:1 VSWR were obtained from Fig. 5 and were used to calculate the pi and c mode parameters from [10]. The parameters were applied in ANA to obtain the response of this coupler from 4.5 to 9.0 GHz. The results are shown in Table III. The phase difference between outputs is 90° at band center but is 93° at the low end and 91° at the high end. The reflection coefficients at each port should be resonant at the band center, 6.75 GHz. Instead, the input and direct port resonance shifted downward to 5 GHz, and the resonance on the coupled port shifted above the upper band edge. The isolation dropped to 31 dB at 4 GHz and only 25 dB at 9.5 GHz.

All these effects are due to the different propagation constants of the pi and c modes and not the fact that the coupler was designed for a 1.2:1 VSWR. This can be seen by comparing the values in Table III to those in Table IV, which were calculated with equal propagation velocities for both modes.

VI. CONCLUSIONS

A simple derivation for the admittance parameters of the three-line coupler by means of a capacitance matrix approach was shown. The relationship between physical dimensions and the capacitances per unit length of the line was used to obtain the required line spacings and widths for any arbitrary coupling ratio. It was shown that, for coupling ratios around 3 dB, the dimensions for a perfectly matched coupler are not always easily realizable, but by allowing a small mismatch the dimensions become more reasonable.

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Letters

Correction to "Coaxial Transmission Lines, Related Two-Conductor Transmission Lines, Connectors, and Components: A U.S. Historical Perspective"

JOHN H. BRYANT, FELLOW, IEEE

While revising and editing the above paper, Fig. 5 was inadvertently duplicated. The result of this oversight can be seen on p. 980, where the illustrations for Figs. 4 and 5 are the same. The intended Fig. 4, with its caption, is shown below.

Manuscript received August 8, 1984.

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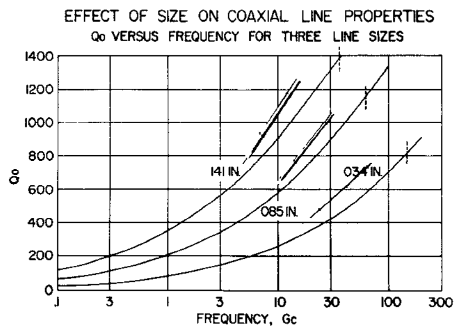

Fig. 4. Unloaded Q of coaxial line versus frequency, for three line sizes. The vertical dashed lines indicate the frequency at which the particular size of transmission line can support a higher order mode.

Patent Abstracts

These Patent Abstracts of recently issued patents are intended to provide the minimum information necessary for readers to determine if they are interested in examining the patent in more detail. Complete copies of patents are available for a small fee by writing: U.S. Patent and Trademark Office, Box 9, Washington, DC 20231.

4,416,505 Nov. 22, 1983

Method for Making Holographic Optical Elements with High Diffraction Efficiencies

Inventor: LeRoy D. Dickson.
Assignee: International Business Machines Corporation.
Filed: Oct. 26, 1981.

Abstract—Production quantities of a multi-element holographic scanner disc are made by optically replicating a silver halide master disc one element at a time in a dichromated gelatin film. The dichromated gelatin film swells during processing. The swell is monitored during production by determining the shift in the angle of the Bragg surfaces within the gel. The angle of the replicating beam for each element is changed from that of the original reference beam to establish a Bragg angle at exposure which will be tilted to the proper angle after swelling in order to maximize the diffraction efficiency of the element at the original reference beam angle

4 Claims, 6 Drawing Figures